Write your name and student number on each page. This exam comprises 5 problems. The total number of points is 40. The final grade is obtained by dividing the number of points by 4.

Problem 1: Research reactor (10 pts)

A research reactor is in the shape of a parallelepiped with a square base of side 5.2 m and a height 6.8 m. The reactor is filled uniformly with a fuel with the properties
$\nu\Sigma_f = 0.0072\ cm^{-1}$, $\nu = 2.45$ and $\Sigma_a = 0.0070\ cm^{-1}$.
The reactor operates steadily at a fission power of 15 MW. The average value of energy per fission is $E_f = 200\ MeV$, and $1\ eV = 1.6 \times 10^{-19}\ J$.

a. Calculate the value of the diffusion coefficient. (2 pts)
b. Calculate the average **and** maximum values of the neutron flux. (4 pts)
c. At what rate is the fuel consumed in the entire reactor (in nuclides/s) **and** at the centre of the reactor (in nuclides/$(cm^3 \cdot s)$)? (4 pts)

Solution

a. The buckling is

$$B^2 = \frac{1}{D}\left(\frac{\nu}{k}\Sigma_f - \Sigma_a\right)$$

From which we deduce the diffusion coefficient D:

$$D = \frac{1}{B^2}\left(\frac{\nu}{k}\Sigma_f - \Sigma_a\right)$$

Since the reactor operates steadily, we have $k = 1$ and the diffusion coefficient becomes:

$$D = \frac{1}{B^2}\left(\nu\Sigma_f - \Sigma_a\right)$$

We now calculate the buckling for a parallelepiped with a square base reactor:

$$B^2 = \left(\frac{\pi}{520}\right)^2 + \left(\frac{\pi}{520}\right)^2 + \left(\frac{\pi}{680}\right)^2 = 9.434 \times 10^{-5}\ cm^{-2}$$

The diffusion coefficient is then

$$D = \frac{1}{B^2}\left(\nu\Sigma_f - \Sigma_a\right) = \frac{1}{9.434 \times 10^{-5}}(0.0072 - 0.0070) = 2.12\ cm$$

b. The maximum flux is at the centre of the reactor

$$\phi(0,0,0) = A\cos\left(\frac{\pi}{520} \times 0\right)\cos\left(\frac{\pi}{520} \times 0\right)\cos\left(\frac{\pi}{680} \times 0\right) = 3.87 \times \frac{P}{VE_f\Sigma_f}$$

$$\phi(0,0,0) = 3.87 \times \frac{15\ MW}{520^2 \times 680 \times 200\ MeV \times \frac{0.0072}{2.45}}$$

$$\phi(0,0,0) = 3.87 \times \frac{15 \times 10^6\, J/s}{520^2 \times 680 \times (200 \times 1.6022 \times 10^{-13}\, J) \times \frac{0.0072}{2.45}}$$
$$= 3.35 \times 10^{12}\ cm^{-2}s^{-1}$$

The total power of the reactor is
$$P = E_f R_f V = E_f \Sigma_f \phi V$$
The average flux is
$$\phi = \frac{P}{E_f \Sigma_f V} = \frac{15 \times 10^6\, J/s}{520^2 \times 680 \times (200 \times 1.6022 \times 10^{-13}\, J) \times \frac{0.0072}{2.45}}$$

$$\phi = 8.66 \times 10^{11}\ cm^{-2}s^{-1}$$

c. The fuel in the reactor is consumed by fissions and neutron captures.
The macroscopic cross section is
$$\Sigma_f + \Sigma_{cap} = \Sigma_a$$

The total fuel consumption in the reactor is
$$\Sigma_a \phi V = 0.0070\ cm^{-1} \times 8.66 \times 10^{11}\ cm^{-2}s^{-1} \times 520^2 cm^2 \times 680\ cm$$

$$\Sigma_a \phi V = 1.11 \times 10^{18}\ s^{-1}$$

The fuel consumption in the middle of the reactor is
$$\Sigma_a \phi(0,0,0) = 0.0070\ cm^{-1} \times 3.35 \times 10^{12}\ cm^{-2}s^{-1} = 2.35 \times 10^{10}\ cm^{-3}s^{-1}$$

Problem 2: Pressurized Water Reactor (8 pts)

a. Explain why a pressurized light water reactor uses enriched fuel, while a pressurized heavy water reactor uses natural uranium. (4 pts)
b. Explain why a reactor with k close to 1 and a negative temperature coefficient of reactivity is inherently stable. (4 pts)

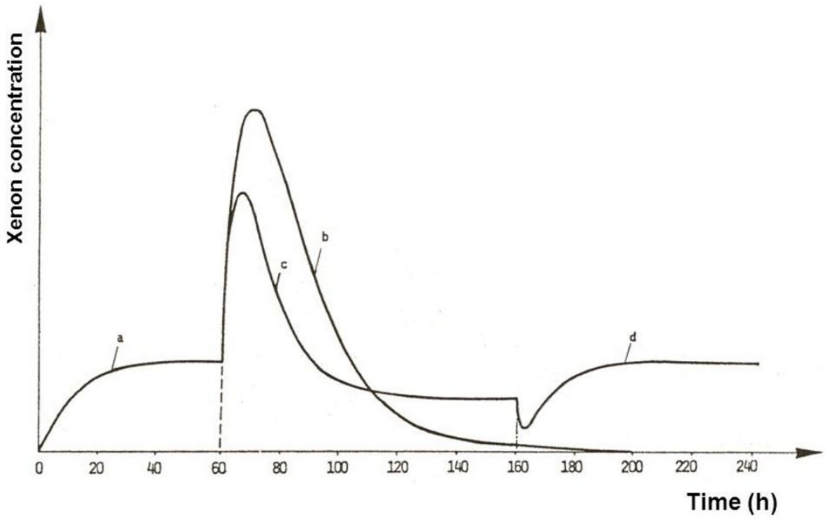
Solution

a. Water has the highest moderator value but has a relatively large absorption for neutrons, which means that we will loose some of them and those will not participate in fission reactions. This can be compensated by enriching the fuel, that is increasing the amount of fissile material (3-5%) ^{235}U. A heavy water reactor uses D2O, which has a low neutron absorption and therefore can use natural uranium.

b. The temperature coefficient of reactivity is
$$\alpha_T = \frac{1}{k}\frac{dk}{dT}$$

In the case where α_T is negative, an increase in temperature results in a decrease in k, which leads to a decrease in power. A decrease in power leads to a decrease in temperature and returns the reactor to its initial state. If the reactor experiences a decrease in temperature, k would increase, and therefore the power as well. So, the reactor once more returns to its original state.

Problem 3: Xenon concentration (8 pts)
The figure below shows the response of the xenon-135 concentration as a function of time to changes in the power level of a reactor. Describe what happens to the xenon concentration and deduce the status of the power level for the curves marked a, b, c and d.



Solution
- The curve a corresponds to the reactor startup, as there is no xenon at the start. The concentration of xenon builds up over time.
- The curve b corresponds to a reactor shutdown. The xenon concentration experiences a buildup because iodine decays faster than xenon. Once $\lambda_I N_I$ goes to zero, xenon flows the radioactive law.
- The curve c corresponds to a reduction of power and a buildup of xenon concentration.
- The curve d corresponds to the power level restored to its initial value. The xenon concentration initially decreases because the burnup is increased at the new higher power level. The xenon concentration then increases to the new equilibrium level for the new power level.

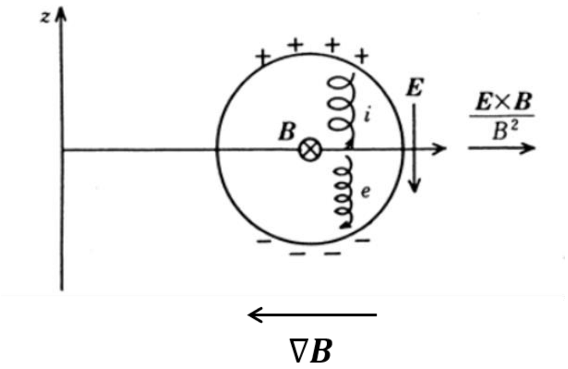
Problem 4: Fusion plasma (8 pts)

Consider a plasma of electrons and positively charged ions in a torus. Toroidal field coils produce a magnetic field $\boldsymbol{B}_\phi$ within the vessel of this torus.

Explain why it is not possible to confine a plasma using only curved magnetic fields. In addition, sketch the circular cross section of the torus and indicate the direction of the magnetic field, the gradient of the magnetic field, and the other quantities necessary to support your explanation.

Solution
A gradient in the magnetic field creates a $\nabla \boldsymbol{B}$ force which drives the positive ions in one transverse direction and the negative electrons in the other. This results in a local charge

separation, which generates a vertical electric field. In addition, the Lorentz force will cause the ions and electrons to drift ($\boldsymbol{E} \times \boldsymbol{B}$) in an outward direction, perpendicular to the major torus axis. The plasma in a simple toroidal filed will drift outward until it strikes the surrounding wall. This is due to the toroidal drift, which is the sum of the curvature and gradient drifts.



In this case there is no radial equilibrium and therefore no plasma confinement.

Problem 5: ITER (6 pts)

a. One of the goals of ITER is to achieve a fusion gain $Q = 10$, i.e. injecting 50 MW in the plasma to produce 500 MW. However, in class we calculated that the engineering gain factor was $Q_E = 1.5$, far from the advertised fusion gain. Explain why the engineering gain factor is so much lower than the physics one.
b. List what you think are the most important goals of ITER, and argue whether the gigantic price tag of ITER (> € 22 billion) is justified or not towards the possibility of building and using fusion reactors in the future.

Solution

a. During operations the electrical consumption of the ITER machine and its facilities should be on the order of 110MW. Therefore, taking this value into account the fusion Q would be 10 and the engineering factor would be 500/110 (i.e., 4.5). We need to divide the engineering gain by 3 because the 500 MW produced by fusion reactions is thermal power, while the 110 MW injected is electrical power and QE would therefore be 1.5.

b. It is important to remember that ITER is a gigantic experiment, not a power plant that will deliver electricity to the grid. While the engineering gain factor is $Q_E = 1.5$, and not the advertised 10, this number is still greater than 1 and net electrical power will be produced by ITER.

The goals of ITER are the following:
Study and test technologies such as heating, control, diagnostics, cryogenics and remote maintenance in an **integrated way.**

Achieve a deuterium-tritium plasma in which the reaction is sustained through internal heating: **burning plasma**, i.e. the heat from the fusion reaction stays within the plasma long enough for the reaction to be sustained for a long duration.

Test **tritium breeding**: demonstrate the feasibility of producing tritium within the vacuum vessel. ITER provides a unique opportunity to test mockup in-vessel **tritium breeding blankets** in a real fusion environment

Demonstrate the **safety characteristics** of a fusion device: demonstrate control of the plasma and fusion reactions with negligible consequences to the environment.